



5.2 The norm induced by the inner product

Douglas Wilhelm Harder, LEL, M.Math.

dwharder@uwaterloo.ca

dwharder@gmail.com





Introduction

- In this topic, we will
 - Define the norm induced by an inner product
 - Make some comments





The induced vector norm

- In any inner product space, if $\mathbf{u} \in \mathcal{V}$

$$\|\mathbf{u}\|_2 = \sqrt{\langle \mathbf{u}, \mathbf{u} \rangle}$$

- After all, note that for $\mathbf{u} \in \mathbf{R}^n$

$$\sqrt{\langle \mathbf{u}, \mathbf{u} \rangle} = \sqrt{\sum_{k=1}^n u_k u_k} = \sqrt{\sum_{k=1}^n u_k^2} = \|\mathbf{u}\|_2$$

- Similarly, for $\mathbf{u} \in \mathbf{C}^n$

$$\sqrt{\langle \mathbf{u}, \mathbf{u} \rangle} = \sqrt{\sum_{k=1}^n u_k^* u_k} = \sqrt{\sum_{k=1}^n |u_k|^2} = \|\mathbf{u}\|_2$$





The induced vector norm

- This course will focus on using the inner product and its properties, and thus, the most common norm we will use is the 2-norm
 - The other properties of the 2-norm follow from the properties of the inner product
- An important variation will use is $\|\mathbf{u}\|_2^2 = \langle \mathbf{u}, \mathbf{u} \rangle$





Summary

- Following this topic, you now
 - Know that the 2-norm is induced by the inner product
 - Understand that this relationship is why we will focus on using the 2-norm





References

- [1] [https://en.wikipedia.org/wiki/Norm_\(mathematics\)](https://en.wikipedia.org/wiki/Norm_(mathematics))





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

<https://www.rbg.ca/>

for more information.





Disclaimer

These slides are provided for the NE 112 *Linear algebra for nanotechnology engineering* course taught at the University of Waterloo. The material in it reflects the authors' best judgment in light of the information available to them at the time of preparation. Any reliance on these course slides by any party for any other purpose are the responsibility of such parties. The authors accept no responsibility for damages, if any, suffered by any party as a result of decisions made or actions based on these course slides for any other purpose than that for which it was intended.

